```
set(auto)
assign(max_seconds, 32700)
        -- – ————————————————————————————
        -- – ——————— AXIOMS ON ORDERED GROUPS
        -- – ————————————————————————————
formula_list(usable)
        -- abelian group axioms
        -- (think that A⊖B=A⊕⊖B    by def.)
    [∀x, ∀y, ∀z | (x⊕y)⊕z=x⊕(y⊕z)]    -- associativity
    [∀x | x⊕e=x]    -- right unit
    [∀x | x⊖x=e]    -- right inverse
    [∀x, ∀y | x⊕y=y⊕x]    -- commutativity
        -- ordering axioms (axioms concerning non-negativeness)
    [∀x, ∀y | nneg(x) & nneg(y) ⟶ nneg(x⊕y)]
    [∀x | nneg(x) ∨ nneg(⊖x)]
    [∀x | nneg(x) & nneg(⊖x) ⟶ x=e]
end_of_list
        -- – ————————————————————————————
        -- – ——————— DEFINITIONAL EXTENSIONS
        -- – ————————————————————————————
formula_list(usable)
    [∀x | nneg(x) ⟶ |x|=x]    -- definition of the absolute value ...
    [∀x | ¬nneg(x) ⟶ |x|=⊖x]    -- ...def.n of the absolute value
    [∀x, ∀y | x≼y ↔ nneg(y⊖x)]    -- def.n of comparison
end_of_list
        -- – ————————————————————————————
        -- – ——————— LEMMAS ————————————
        -- – ————————————————————————————
formula_list(usable)    -- laws concerning the "leq" relation ≼
        -- -- statements A1, Ba, Bb, B below proved without definitional extensions
    [∀x, ∀y, ∀z | x⊕y=x⊕z ⟶ y=z]    -- A1: cancellation law
        -- [∀x, ∀y | x⊕⊖y⊕(y⊕⊖x)=e]    -- Ba
        -- [∀x, ∀y | x⊕⊖y⊕⊖(x⊕⊖y)=e]    -- Bb
    [∀x, ∀y | ⊖(x⊖y)=y⊖x]    -- B (from A1, Ba, Bb alone)
    [∀x, ∀y | x≼y ∨ y≼x]    -- C: totality
    [∀x | x≼x]    -- D: reflexivity
    [∀x, ∀y, ∀z | x≼y & y≼z ⟶ x≼z]    -- E: transitivity
    [∀x, ∀y, ∀z | x≼y & x≠y & y≼z ⟶ x≠z]    -- E1: transitivity
    [∀x, ∀y, ∀z | x≼y & y≼z & y≠z ⟶ x≠z]    -- E2: transitivity
    [∀x, ∀y, ∀z | x≼y ⟶ x⊕z≼y⊕z]    -- F: additivity
    [∀x, ∀y, ∀z | x⊕z=y⊕z ⟶ x=y]    -- A2: cancellation law
    [∀x, ∀y, ∀z | x≼y & x≠y ⟶ x⊕z≠y⊕z]    -- F1, strict additivity
end_of_list
        -- – ————————————————————————————
        -- – ————————————————————————————
        -- – ————————————————————————————
```

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formula_list(usable)      --  laws concerning the operations "abs" and "nneg"
    [∀x | |x⊖x|=e]      --  1
    [∀x | x≼|x|]      --  2
    [∀x | |(|x|)|=|x|]      --  3
    [∀x | |x|=e ↔ x=e]      --  4
    [∀x | |⊖x|=|x|]      --  5
    [∀x, ∀y | x⊕y≼|x|⊕|y|]      --  6
    [∀x, ∀y | nneg(x⊕y) → |x⊕y|≼|x|⊕|y|]      --  7a
    [∀x, ∀y | ¬nneg(x⊕y) → nneg(⊖x⊖y)]      --  7b
    [∀x, ∀y | ¬nneg(x⊕y) → |⊖x⊖y|≼|⊖x|⊕|⊖y|]      --  7c (proved without earlier laws on "leq")
    [∀x, ∀y | ¬nneg(x⊕y) → |x⊕y|≼|x|⊕|y|]      --  7d
    [∀x, ∀y | |x⊕y|≼|x|⊕|y|]      --  7
    [∀x, ∀y, ∀z | x⊕z=x⊖y⊕(y⊕z)]      --  8a
    [∀x, ∀y, ∀z | |x⊖z|≼|x⊖y|⊕|y⊖z|]      --  8 (proved without the axioms)
    [∀x, ∀y | ¬nneg(x) → x≼|y| & x≠|y|]      --  9
    [∀x, ∀y | nneg(y) → x⊖y≼x⊕y]      --  10
    [∀x, ∀y | nneg(x) & ¬nneg(y) → ||x|⊖|y||≼|x⊖y|]      --  11a
    [∀x, ∀y | nneg(x) & nneg(y) → ||x|⊖|y||=|x⊖y|]      --  11b
    [∀x, ∀y | ¬nneg(x) & ¬nneg(y) → ||⊖x|⊖|⊖y||=|⊖x⊕y|]      --  11c
        --  to prove the next lemma, it turned out useful to temporarily inhibit:
        --  * all axioms save commutativity;
        --  * all definitional extensions;
        --  * all of the laws concerning "leq" save D (reflexivity) and B;
        --  * all of the laws concerning "abs" and "nneg" save 5 and 11a–11c
    [∀x, ∀y | ||x|⊖|y||≼|x⊖y|]      --  11
    [∀x, ∀y | |x|⊖||y|⊖|x||≼|y|]      --  12
end_of_list
        --  – ————————————————————————————————
```