A quick introduction to formative processes

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Definition 1 *A well-ordered family* $\Pi, \prec$ *of sets is called a* SUPERPARTITION *if the following conditions hold:*

- $(\forall\, \Sigma \in \Pi)(\forall\, \sigma \in \Sigma)(\sigma \neq \emptyset)$*;*
- $(\forall\, \Sigma_0, \Sigma_1 \in \Pi)(\forall\, \sigma \in \Sigma_0)(\forall\, \tau_1, \tau_2 \in \Sigma_1)(\sigma \ni\in \tau_1 \;\&\; \sigma \ni\in \tau_2 \;\rightarrow \tau_1 = \tau_2)$,
  viz., distinct elements of a set in Π *cannot intersect the same element of a set in* Π*;*
- $\prec$ *is endowed of a* maximum *in* Π*;*
- *for all* $\Lambda \in \Pi$ *which* does not have immediate predecessor *in* Π*,*

$$\Lambda = \left\{ \bigcup \{q : \Gamma \prec \Lambda,\; q \in \Gamma \mid q \ni\in p\} : \Gamma \prec \Lambda,\; p \in \Gamma \right\} .$$

A superpartition whose maximum element is a finite *set, and in which every* Σ *endowed with* immediate *predecessor* Γ *satisfies, for some* $\Delta \subseteq \Gamma$*, the inclusions*

$$\bigcup \Gamma \subseteq \bigcup \Sigma \subseteq \mathcal{P}^*(\Delta) \cup \bigcup \Gamma \,,$$

is called a (FORMATIVE) PROCESS*.* □

It turns out readily that every element Σ of a superpartition is a partition; moreover, the maximum in a formative process is a transitive partition. Since any well-ordered set is order-isomorphic to a unique ordinal number $\xi, <$ (where $< =_{\mathrm{Def}} \in$), we can uniquely represent any superpartition $\Pi, \prec$ in the form of a $(\xi+1)$-sequence $(\Sigma_\mu)_{\mu \leqslant \xi}$ of partitions. Then, for all $\sigma \in \bigcup \Pi$, $\Gamma \in \bigcup \mathcal{P}[\Pi]$, and $\mu \leqslant \xi$ we can unambiguously put

$$\begin{array}{lll} \sigma^{(\mu)} & =_{\mathrm{Def}} & \text{the } \tau \in \Sigma_\mu \text{ for which } \sigma \ni\in \tau \text{ if any exists, else } \emptyset; \\ \Gamma^{(\mu)} & =_{\mathrm{Def}} & \{\gamma^{(\mu)} : \gamma \in \Gamma\}; \\ \sigma^{(\bullet)} & =_{\mathrm{Def}} & \sigma^{(\xi)}, \qquad \Gamma^{(\bullet)} \;=_{\mathrm{Def}}\; \Gamma^{(\xi)}. \end{array}$$

(Thus, for example, $\Sigma_\xi = \{\sigma^{(\xi)} : \sigma \in \bigcup \Pi\} = \{\sigma^{(\bullet)} : \sigma \in \bigcup \Pi\}$ in a formative process.)

Notice that the function $\sigma \mapsto \sigma^{(\mu)}$ is *injective* when restricted to the set $\{\sigma \in \Sigma_\nu \mid \sigma^{(\mu)} \neq \emptyset\}$, for all $\mu, \nu \leqslant \xi$. In the case of a formative process, it turns out easily that Σ_ξ is a transitive partition; accordingly, the function $\left\{\sigma^{(\bullet)}\right\}_{\sigma \in \Sigma_\nu}$ will be injective for all $\nu \in \xi$ (and also, obviously, for $\nu = \xi$, in which case $\sigma \overset{\iota_{\Sigma_\xi}}{\mapsto} \sigma^{(\bullet)}$).

Quite often, in the case of a formative process we will indicate by $(\bullet)$ a bijection $q \mapsto q^{(\bullet)}$ from the places $\mathcal{P}$ of a colored board to Σ_ξ, and will also indicate by $q^{(\mu)}$ the block $\left(q^{(\bullet)}\right)^{(\mu)}$ of Σ_μ, for all $q \in \mathcal{P}$ and $\mu \leqslant \xi$.